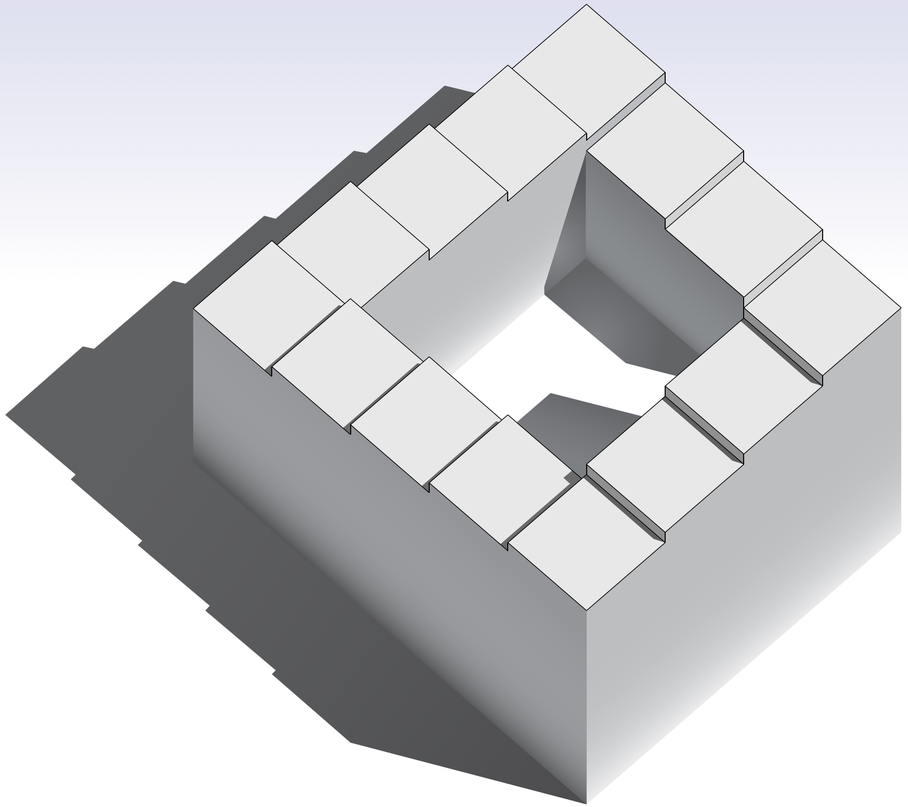
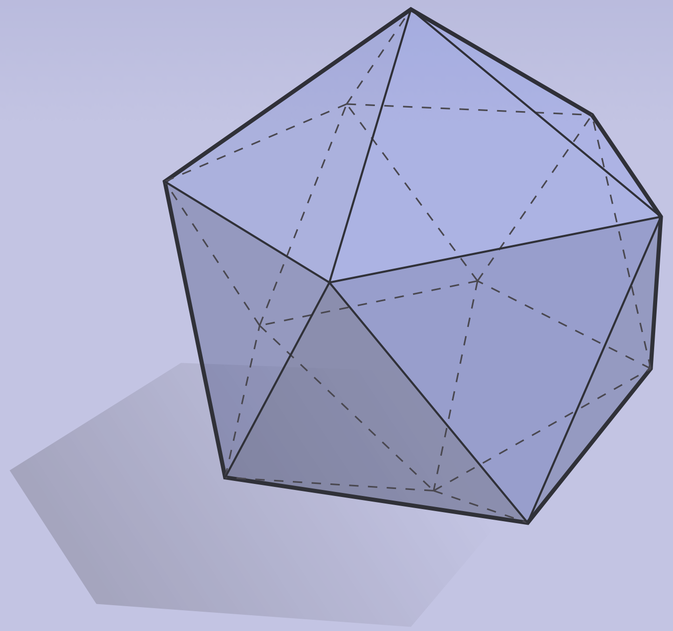


Discrete Differential Geometry: An Applied Introduction

Keenan Crane • CMU 15-458/858

Lecture 0:

Administrivia



Discrete Differential Geometry: An Applied Introduction

Keenan Crane • CMU 15-458/858

Online Lectures

- Right now we're online!
- Hopefully we'll be moving to in-person soon…
- For now we'll do online lectures through **ohyay** (not Zoom)
  - https://ohyay.co/s/ddg2022
- It's recommended that you use Chrome 74+, Firefox 60+, or Safari 12+.

Getting Connected

Three ways to connect with the class:

1. **Course webpage.** Assignments, readings, lecture slides/videos.

   `geometry.cs.cmu.edu/ddg`

2. **Piazza.** Announcements & persistent discussion.

   `piazza.com/cmu`

3. **Discord.** Real-time chat with classmates/TAs/instructor.

   (see Piazza for invite link)

Course Web Page

- All course information is spelled out in detail on course webpage*:

`http://geometry.cs.cmu.edu/ddg`

- Assignments, readings, lecture slides, etc. (& you can comment)
- Register account from link at end of menu (*Andrew email only!*):

*If you're getting `DDGSpringX` for X < 2022, clear your browser cache!

Assignment -1

- You already have your first "assignment":
- Post your favorite mathematical formula/equation on the web page
  - Explain to your classmates what it means/why you like it
  - Good chance to try TeX if you never have! :-)

Assignment -1: Favorite Formula

Part of your course grade is determined by participation, which can include both in-class participation as well as discussion here on the course webpage. Therefore, your first assignment is to:

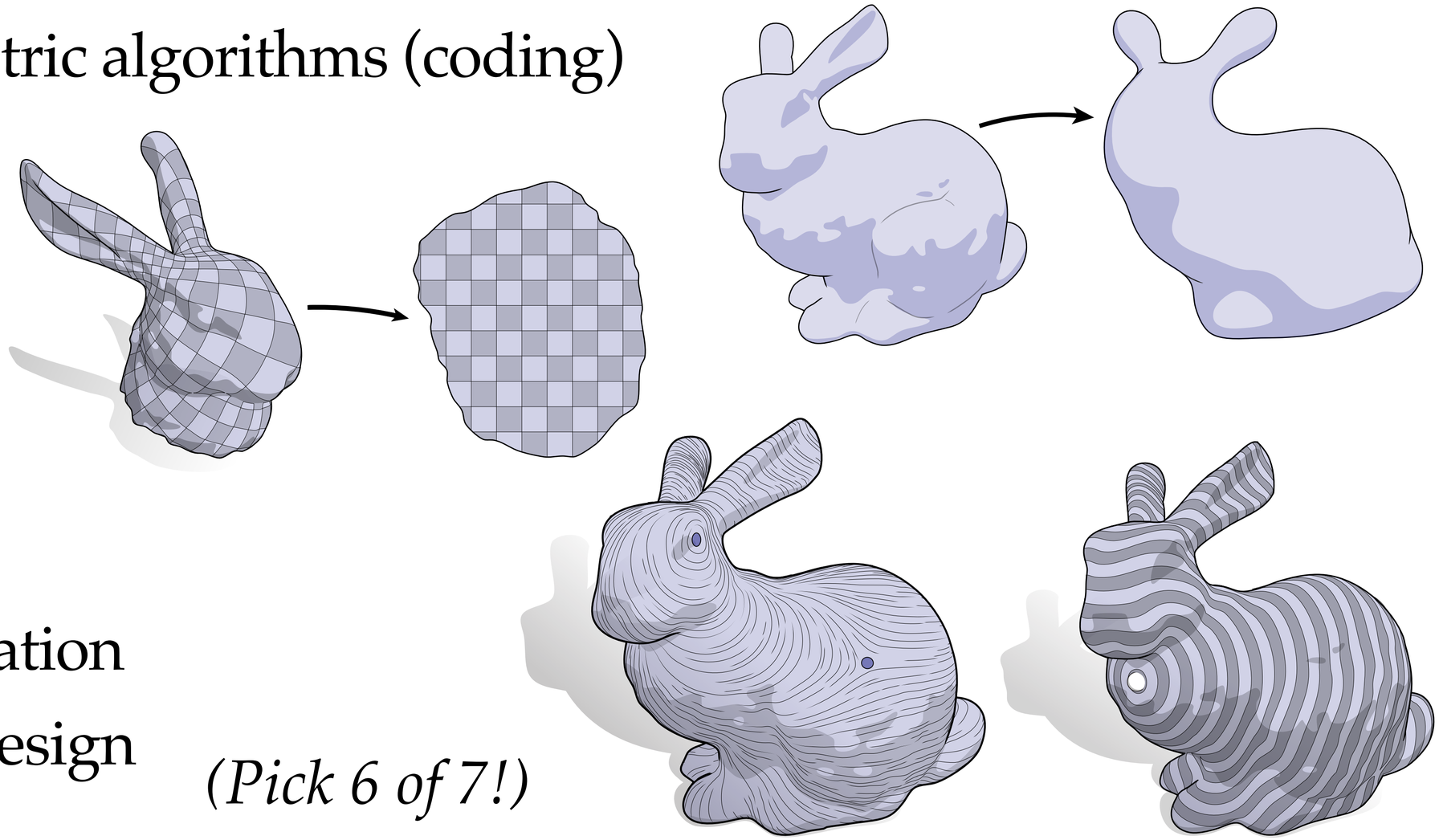
1. create an account on the course webpage (you **must** use your Andrew email address, so we can give you participation credit!),
2. sign up for Piazza and Discord,
3. read carefully through the Course Description and Grading Policy, and
4. leave a comment on this post containing your favorite mathematical formula (see below).

To make things interesting, your comment should include a description of your favorite mathematical formula typeset in LaTeX. If you don't know how to use LaTeX this is a great opportunity to learn — a very basic introduction can be found here. (And if you don't have a favorite mathematical formula, this is a great time to pick one!)

(P.S. Anyone interested in hearing about some cool *"favorite theorems"* should check out this podcast.)

Assignments A0–A6

- **Derive** geometric algorithms from first principles (pen-and-paper)
- **Implement** geometric algorithms (coding)
  - Discrete surfaces
  - Exterior calculus
  - Curvature
  - Smoothing
  - Parameterization
  - Distance computation
  - Direction Field Design



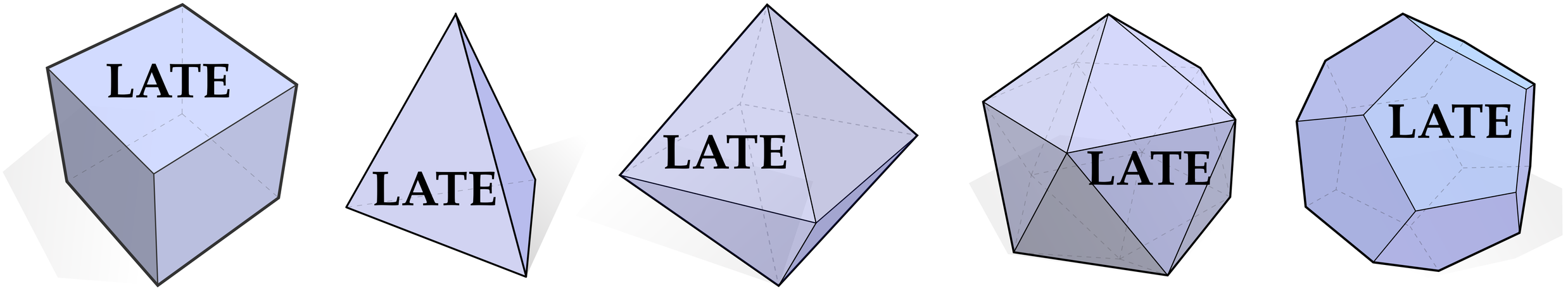
(Pick 6 of 7!)

Homework Submission

- All homework must be submitted digitally via Canvas
  - Source files in a **single zip file** called `solution.zip`
  - Written exercises in a **single PDF file** called `exercises.pdf`
    - Either typeset (e.g., LaTeX) or scans/photos of written work.
    - Convert images to PDF using Preview (Mac) or imagetopdf.com
  - Submit via Canvas
  - Will receive written feedback via email as marked-up PDF
    - Will *try* to have AN graded by the time your turn in A(N+1)

Late Policy

- Assignments due at 5:59:59pm on due date (Eastern time zone)
- Can use **five late days** throughout semester (incl. job interviews!)
  - Must indicate which late day you're using by putting one of five "Latonic" solids on your submission (draw by hand or include PDF):

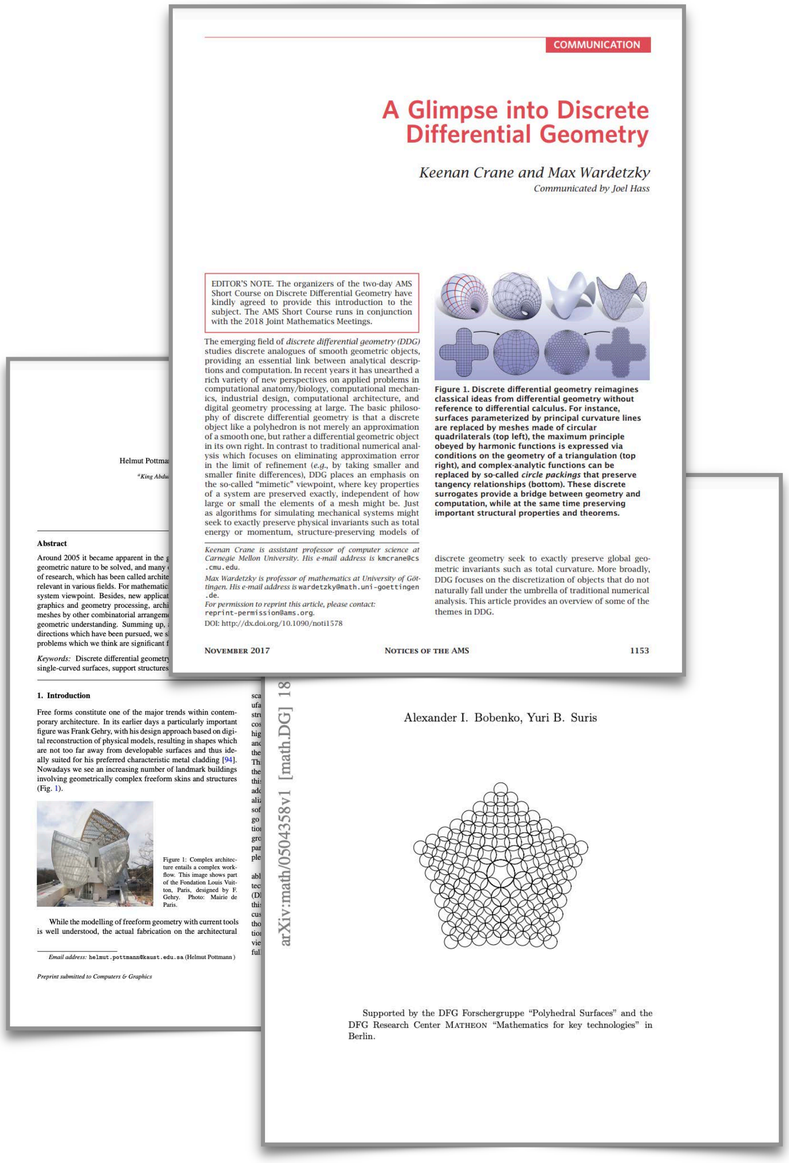


1 **2** **3** **4** **5**

- *All subsequent late work will receive a zero!*

Readings R1—R9

- Additional readings from our course notes, other notes, and/or research papers to enrich perspective, help w/ assignments.
- *Don't worry if you don't understand everything! :-)*
- Will write & submit a short (~1-paragraph) summary for each reading, plus something you didn't understand/wanted to know more about.
- See course webpage for details of hand-in process.



COMMUNICATION

A Glimpse into Discrete Differential Geometry

Keenan Crane and Max Wardetzky

Communicated by Joel Hass

EDITOR'S NOTE. The organizers of the two-day AMS Short Course on Discrete Differential Geometry have kindly agreed to provide this introduction to the subject. The AMS Short Course runs in conjunction with the 2018 Joint Mathematics Meetings.

The emerging field of *discrete differential geometry (DDG)* studies discrete analogues of smooth geometric objects, providing an essential link between analytical descriptions and computation. In recent years it has unearthed a rich variety of new perspectives on applied problems in computational anatomy/biology, computational mechanics, industrial design, computational architecture, and digital geometry processing at large. The basic philosophy of discrete differential geometry is that a discrete object like a polyhedron is not merely an approximation of a smooth one, but rather a differential geometric object in its own right. In contrast to traditional numerical analysis which focuses on eliminating approximation error in the limit of refinement (*e.g.*, by taking smaller and smaller finite differences), DDG places an emphasis on the so-called "mimetic" viewpoint, where key properties of a system are preserved exactly, independent of how large or small the elements of a mesh might be. Just as algorithms for simulating mechanical systems might seek to exactly preserve physical invariants such as total energy or momentum, structure-preserving models of discrete geometry seek to exactly preserve global geometric invariants such as total curvature. More broadly, DDG focuses on the discretization of objects that do not naturally fall under the umbrella of traditional numerical analysis. This article provides an overview of some of the themes in DDG.

Figure 1. Discrete differential geometry reimagines classical ideas from differential geometry without reference to differential calculus. For instance, surfaces parameterized by principal curvature lines are replaced by meshes made of circular quadrilaterals (top left), the maximum principle obeyed by harmonic functions is expressed via conditions on the geometry of a triangulation (top right), and complex-analytic functions can be replaced by so-called *circle packings* that preserve tangency relationships (bottom). These discrete surrogates provide a bridge between geometry and computation, while at the same time preserving important structural properties and theorems.

Keenan Crane is assistant professor of computer science at Carnegie Mellon University. His e-mail address is kmcrane@cs.cmu.edu.

Max Wardetzky is professor of mathematics at University of Göttingen. His e-mail address is wardetzky@math.uni-goettingen.de.

DOI: http://dx.doi.org/10.1090/noti1578

Helmut Pottma

Abstract

1. Introduction

Free forms constitute one of the major trends within contemporary architecture. In its earlier days a particularly important figure was Frank Gehry, with his design approach based on digital reconstruction of physical models, resulting in shapes which are not too far away from developable surfaces and thus ideally suited for his preferred characteristic metal cladding [94]. Nowadays we see an increasing number of landmark buildings involving geometrically complex freeform skins and structures (Fig. 1).

Figure 1: Complex architecture entails a complex workflow. This image shows part of the Fondation Louis Vuitton, Paris, designed by F. Gehry. Photo: Mairie de Paris.

While the modelling of freeform geometry with current tools is well understood, the actual fabrication on the architectural

Email address: helmut.pottmann@kaust.edu.sa (Helmut Pottmann)

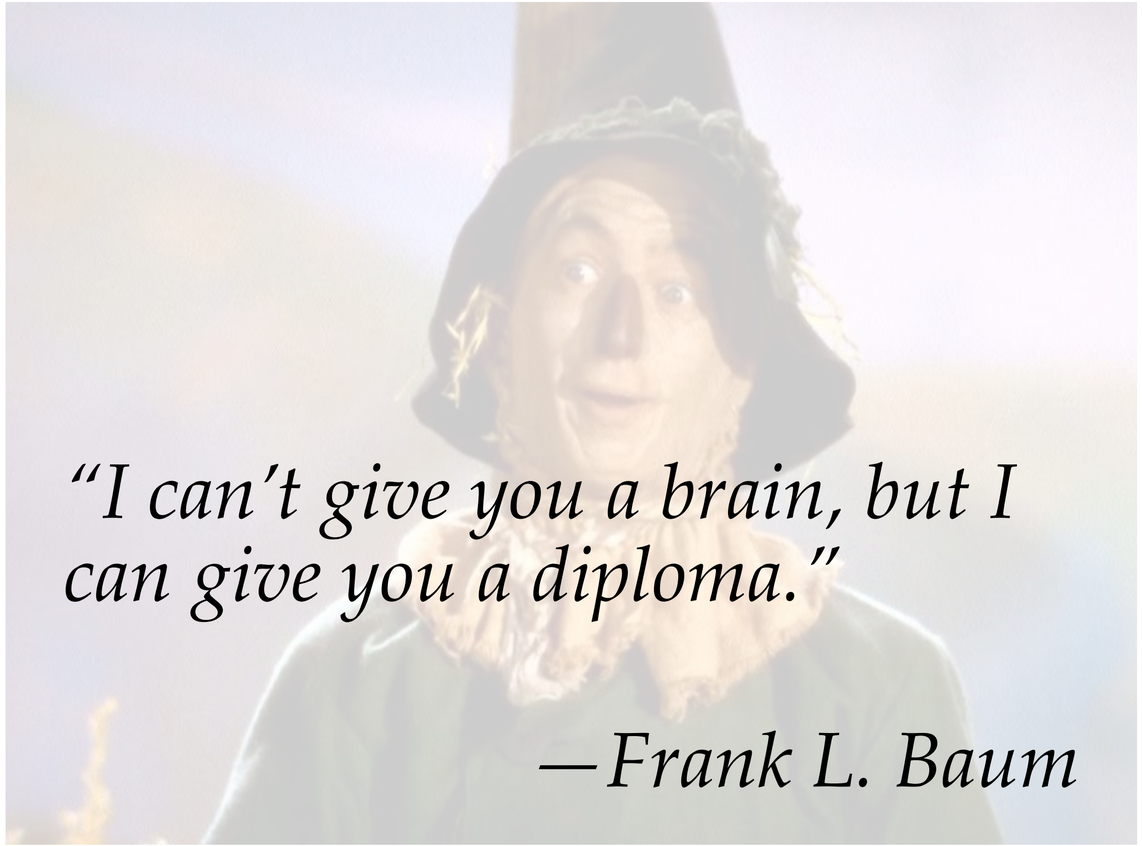
Preprint submitted to Computers & Graphics

Alexander I. Bobenko, Yuri B. Suris

Supported by the DFG Forschergruppe "Polyhedral Surfaces" and the DFG Research Center MATHEON "Mathematics for key technologies" in Berlin.

Grade Breakdown

- **Assignments** – 90% (pick **6** out of 7*)
  - (15%) A0: *Combinatorial Surfaces*
  - (15%) A1: *Exterior Calculus*
  - (15%) A2: *Normals & Curvature*
  - (15%) A3: *Surface Fairing*
  - (15%) A4: *Surface Parameterization*
  - (15%) A5: *Geodesic Distance*
  - (15%) A6: *Direction Field Design*
- **Participation** – 10%
  - (5%) – in-class/web participation
  - (5%) – reading summaries/questions



**Complete 7th assignment for up to 15% extra credit.*

Lectures

- Lectures will be in-class (/online at beginning of semester)
- There are already video recordings for all lectures on YouTube:

youtube.com/keenancrane

(Playlists ➔ Discrete Differential Geometry)

- Watch if you're sick, traveling, etc.… but otherwise, please come to class!
  - might also finish lecture on video, to make more time for in-class questions
  - helps us stick to the lecture & assignment schedule, rather than "bleeding over"
- **There will not be *additional* recordings of this semester's live lectures**
  - Content is *identical* recorded lectures; administrivia via Piazza
  - You can't ask live questions while watching YouTube :-)

Speed-Running The Course

Common question from students:

"When will X be posted?"

Answer:

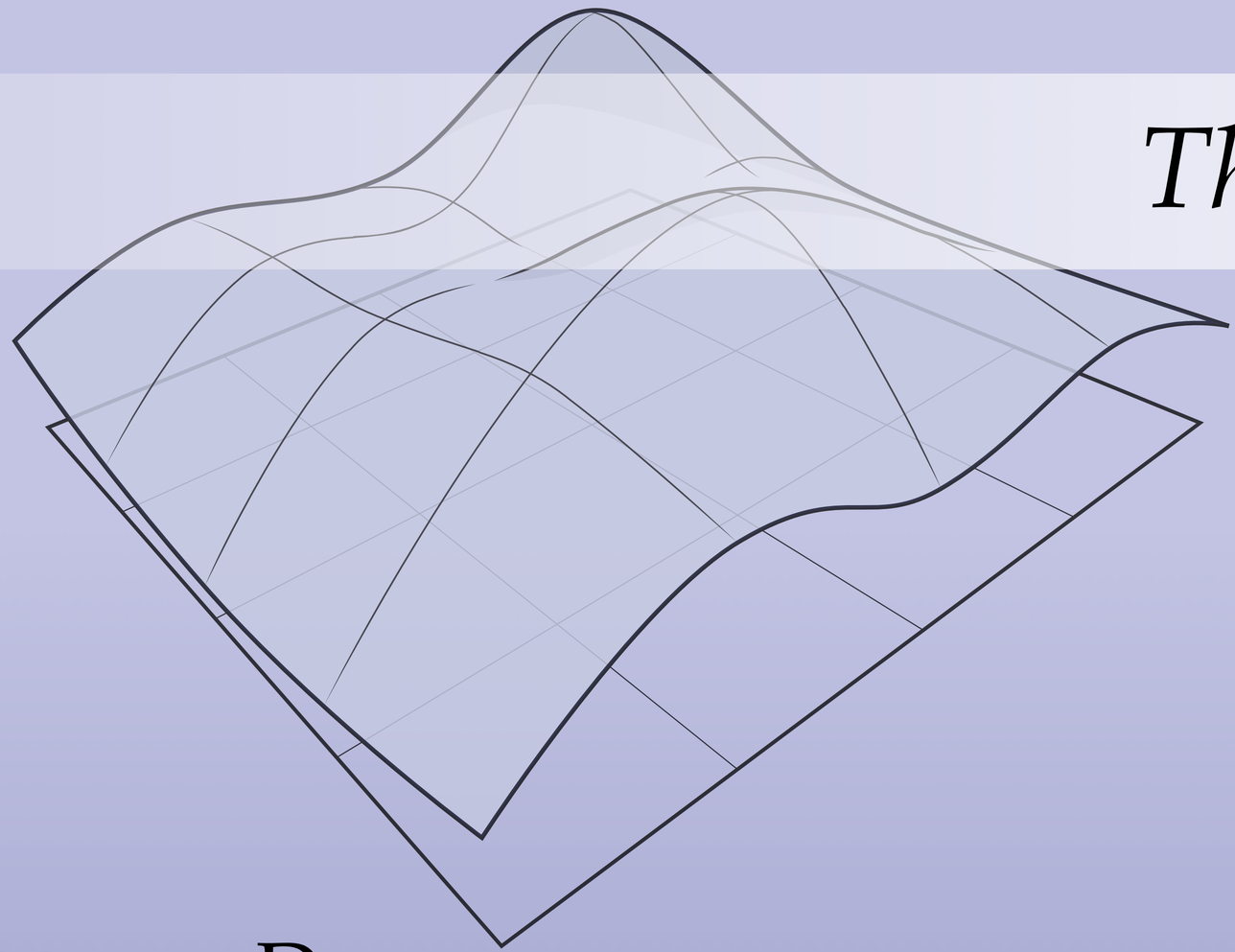
"Everything is already posted now!"

You never have to wait to get started! More precisely:

- All lectures are available ahead of time on YouTube
- All assignment due dates are in the CALENDAR tab
- The code skeleton for A0 is actually a skeleton for A0–A6
- Apart from due dates, assignments are identical to Spring 2021:

`https://brickisland.net/DDGSpring2021/`

Thanks!



Discrete Differential Geometry: An Applied Introduction

Keenan Crane • CMU 15-458/858B • Fall 2017