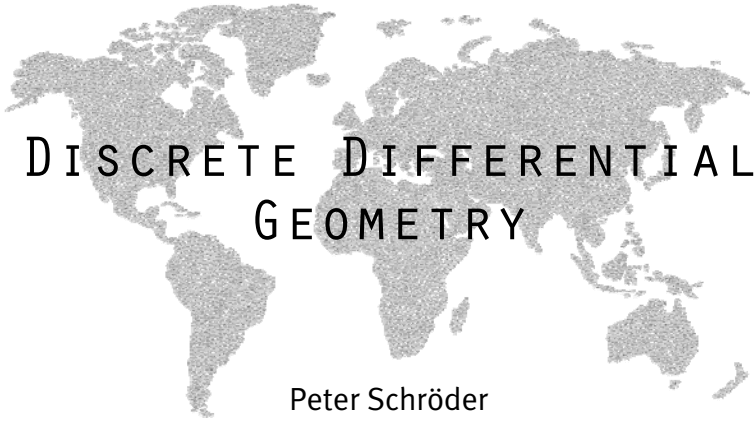
Discrete Differential Geometry

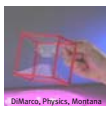


Peter Schröder
with help from Eitan Grinspun, Mathieu Desbrun and the rest of the DDG crew

ps@cs.caltech.edu

A Bit of History

Geometry is the key!



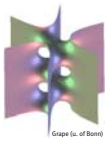
- studied for centuries
  - Cartan, Poincaré, Lie, Hodge, de Rham, Gauss, Noether,...
- mostly differential geometry
  - differential and integral calculus

Study of invariants and symmetries

Differential Geometry

Why do we care?

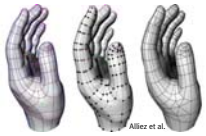
- geometry of surfaces

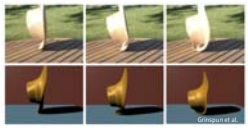


- mothertongue of physical theories

$$E = \int_S \alpha + \beta(H - H_0)^2 + \gamma K \, dA$$

- computation: simulation/processing





Getting Started

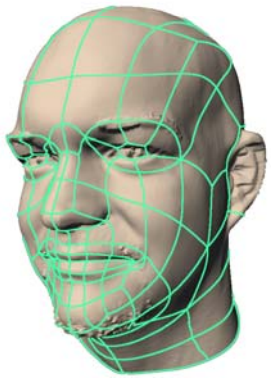
How to apply DiffGeo ideas?

- surfaces as collections of samples
  - and topology (connectivity)
- apply continuous ideas
  - BUT: setting is discrete
- what is the right way?
  - discrete vs. discretized

Discretized

Build smooth manifold structure

- collection of charts
  - mutually compatible on their overlaps
- form an atlas
- realize as smooth functions
  - differentiate away…



Discretization of Eqs

Observation

- be careful

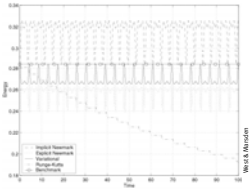
$$M\ddot{q} + C\dot{q} + Kq = f$$

?

$$m\frac{q_{i-1} - 2q_i + q_{i+1}}{\Delta t^2} + c\frac{q_{i+1} - q_i}{\Delta t} + kq_{i+1} = f_i$$

- structure may not be preserved

$$L(q, \dot{q}) = \frac{1}{2}\dot{q}M\dot{q} - V(q)$$



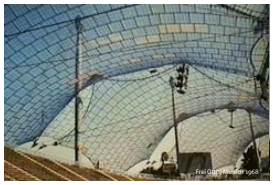
Discrete Geometry

Basic tool

- differential geometry
  - metric, curvature, etc.

Discrete realizations

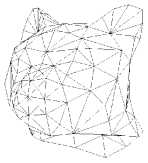
- "meshes"
  - computational geom.
  - graph theory



Discrete Diff. Geometry

Building from the ground up

- discrete geometry is the given
  - meshes: triangles, tets
  - more general: cell complex



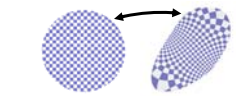
- how to do calculus?
  - preserve crucial properties

$$\int_a^b f'(x)dx = f(b) - f(a)$$

Discr. Diff. Geometry

Building from the top down

- high level theorems
  - Riemann mapping
  - Willmore energy
  - Hamilton's prcple.



$f : \Omega_1 \subset \mathbb{C} \rightarrow \Omega_2 \subset \mathbb{C}$

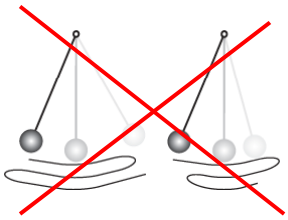
$\int_S (\kappa_1 - \kappa_2)^2 dA$

$$0 = \delta_q \int_0^T L(q(t), \dot{q}(t)) dt$$

Is It Much Better?

Magic happens

- much more robust



Is there a recipe?

- yes, but several ones…
- discrete var. principle

Discrete from the start