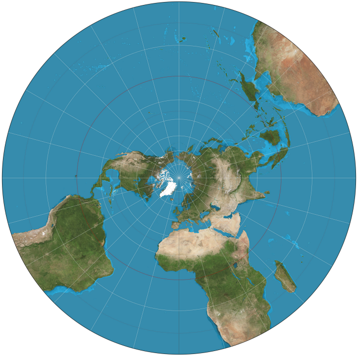
On the Conformal Maps of Triangle Linkages

Anonymous

April 26, 2016
15-869J course project

Conformal map



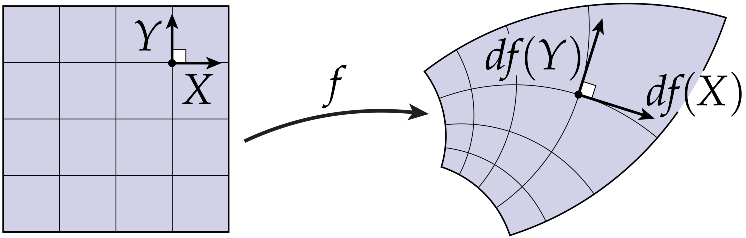
https://upload.wikimedia.org/wikipedia/commons/f/f4/Mercator_projection_SW.jpg and https://upload.wikimedia.org/wikipedia/commons/a/a6/Stereographic_projection_SW.JPG. "By Strebe (Own work) [CC BY-SA 3.0 (http://creativecommons.org/licenses/by-sa/3.0)], via Wikimedia Commons"

Smooth Setting: Angular Definition

$f : M \to \mathbb{C}$ is *conformal* if f has nonvanishing derivative and

$$df(\mathcal{J}X) = i df(X)$$

for all tangent vectors X.

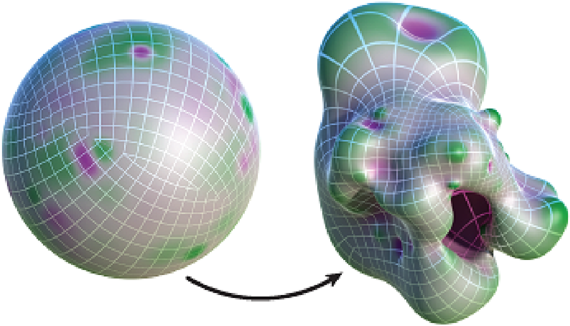


Source: Crane's 15-869 Spring 2016 Slides

Smooth Setting: Metric Scaling Definition

g and $\tilde{g}$ are metrics of a Riemannian Manifold M.

$$g = e^{2\phi}\tilde{g}$$



Source: Crane, et.al. [2011]

Smooth Setting: Conformal Energy

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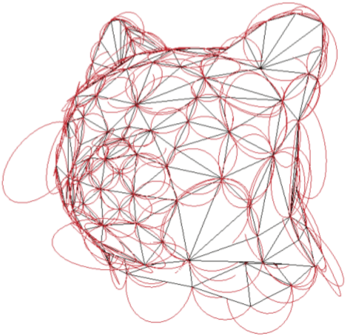
subject to normalizing constraints

$$\sup_{p \in M} |f(p)| = 1$$
$$\langle\langle f(p), 1 \rangle\rangle_M = 0$$

Discretization: Previous Methods

Angles/Circle Packing



Kharevych, et.al. [2006]

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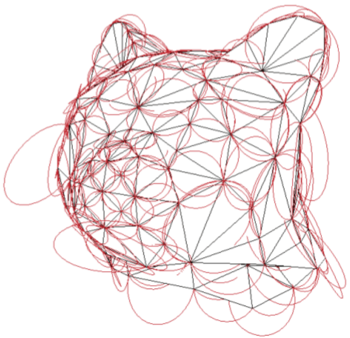
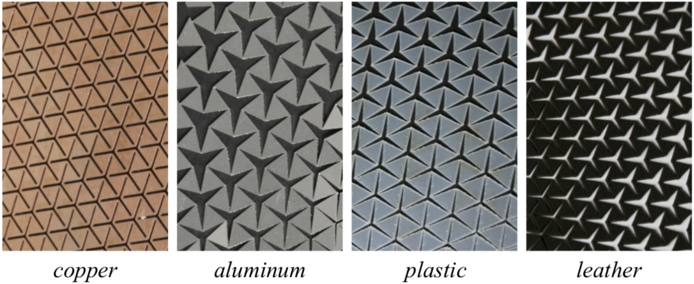


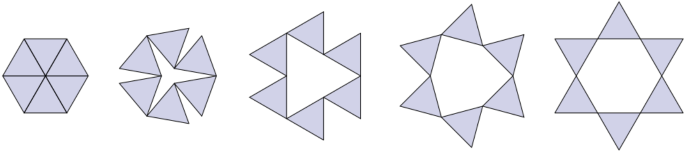
Kharevych, et.al. [2006]

Scaling/Cross Ratios

Springborn, et.al. [2008]

Discretization: Triangular Linkage





Source: Konakovic, et.al. [2016]

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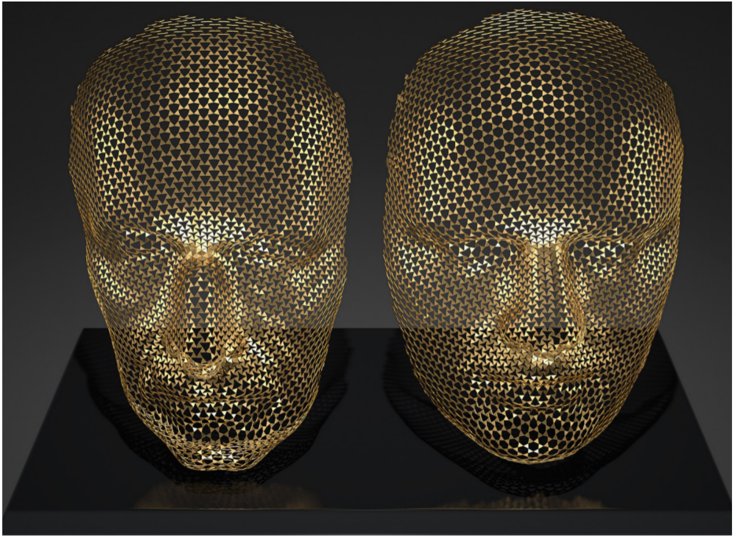

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Implementation

Theoretical Goal

Come up with a (partial) theory of DDG for triangular linkages which explains the conformal structure.

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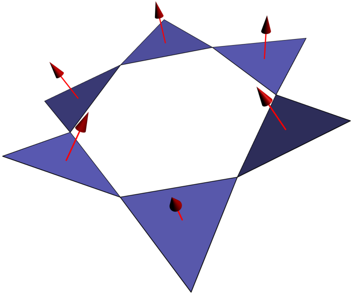
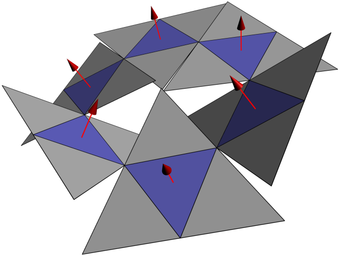
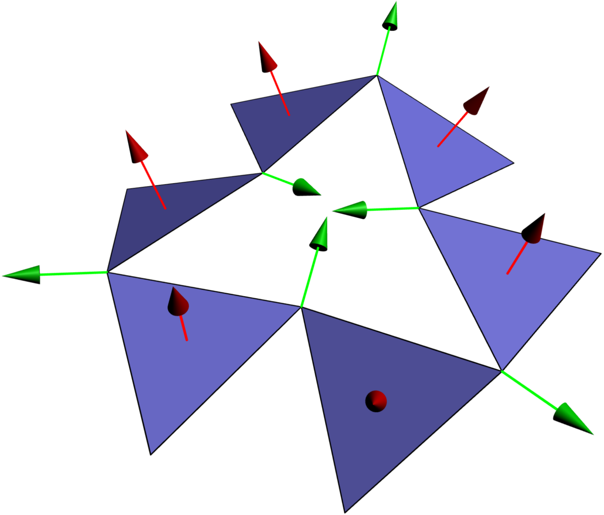




Polthier and Wardetzky

Implementation



Defining Discrete Conformal Maps

Chalkboard.

Thank you!